

Gen Relativ Gravit (2010) 42:1953–1967
DOI 10.1007/s10714-010-0974-8

RESEARCH ARTICLE

Effective field theory analysis of the self-interacting chameleon

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Received: 22 April 2009 / Accepted: 18 March 2010 / Published online: 6 April 2010
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Abstract We analyse the phenomenology of a self-interacting scalar field in the context of the chameleon scenario originally proposed by Khoury and Weltman. In the absence of self-interactions, this type of scalar field can mediate long range interactions and simultaneously evade constraints from violation of the weak equivalence principle. By applying to such a scalar field the effective field theory method proposed for Einstein gravity by Goldberger and Rothstein, we give a thorough perturbative evaluation of the importance of non-derivative self-interactions in determining the strength of the chameleon mediated force in the case of orbital motion. The self-interactions are potentially dangerous as they can change the long range behaviour of the field. Nevertheless, we show that they do not lead to any dramatic phenomenological consequence with respect to the linear case and solar system constraints are fulfilled.

Keywords Chameleon · Effective field theory · Scalar-tensor gravity

1 Introduction

Despite the well established observation that gravitation and electromagnetism are the only long-range forces, the existence of a spin-0 field propagating an additional long-range force was conjectured long ago [1, 2]. Since there is no experimental evidence for such a “fifth” force, there are basically two ways for a scalar particle to exist while

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maintaining the absence of an extra long-range force. This scalar field would have to be either weakly coupled to matter, or massive enough for its interactions to be effectively short-ranged, since allowing a massless scalar field to couple to ordinary matter with gravitational strength would lead to a stark violation of the *equivalence principle*. The weak equivalence principle (WEP) is said to be violated if the coupling depends on parameters other than the mass of the particle to which it couples. The WEP states that [3]

If an uncharged test body is placed at an initial event in space-time and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition.

The standard way to maintain a long-ranged force due to an extra scalar field is to suppress its coupling to matter. The task is not straightforward, however, as scalar fields tend to couple to matter with gravitational strength, see [4,5] for ideas based on symmetry principles to suppress scalar couplings. Otherwise, the presence of light scalar fields, or *moduli*, with run-away potentials is ubiquitous in super-gravity constructions [6]. Their stabilisation (i.e. by giving them a mass) turns out to be a formidable task and has only been achieved in specific constructions [7].

A novel mechanism was suggested [8,9] to suppress long-range interactions mediated by a scalar field while allowing its fundamental dimensionless coupling to ordinary matter be of order unity. This mechanism consists of endowing the scalar field with a mass that depends on local matter density. Such a scalar field is called the *chameleon* for its ability to evade detection by changing its aspect in different environments. In theory, the chameleon can acquire a large enough mass in the neighborhood of the laboratory, allowing it to escape detection and searches for violation of the equivalence principle. It remains effectively massless in almost empty space, for instance in interplanetary space, so that it can propagate to astrophysical or cosmological distances. It turns out that the chameleon violates the WEP.

The equivalence principle has been accurately tested. Experiments testing differential relative acceleration of massive objects like the earth and the moon towards the sun verify the equivalence principle to less than 10^{-13} [3]. Better limits still are expected from forthcoming satellite experiments [10,11]. Moreover, precision tests of gravity provide strong constraints on the post-Newtonian parameters that are used to measure possible deviations from standard General Relativity, which translate into constraints for *scalar-tensor* theories of gravity and the corresponding *linear* coupling of the extra scalars to matter. For the Cassini experiment in particular, Bertotti et al. [12] measurement of the Shapiro time-delay in the solar system gives a constraint on the post-Newtonian parameter γ which provides the strongest bound [3] on scalar-tensor theories endowed with the chameleon mechanism. For this reason, we consider the chameleon model for bodies with weak self-gravity (unlike neutron stars and black holes).

Several models of modified gravity can actually be rewritten in terms of scalar-tensor theories [13], for which the same strict experimental bounds exist. This is the case for DGP models [14], massive gravity [15–18], and $f(R)$ theories of gravity, where the latter require the chameleon mechanism in order to be phenomenologically viable [19,20].

The chameleon was originally suggested in a cosmological context and has been exploited as a quintessential field to give a description of dark energy, see [21, 22] for original suggestions to model dark energy with a scalar field rolling down a flat potential. In this article, however, we do not wish to elaborate on the cosmological implications of the chameleon, but rather point out a theoretical issue that is phenomenologically relevant for the chameleon whenever third or higher order *non-derivative* self-interactions of scalar fields are taken into account. Self-interactions were already considered, either by taking into account the ϕ^4 self-interaction [23, 24] (where ϕ denotes the scalar field) or by considering a generic type of potential [25, 26]. We show that qualitatively different results are possible if ϕ^3 interactions are taken into account, as derived in [27], even if such modifications are quantitatively negligible for most ranges of parameter values. Our analysis is an application of the effective field theory approach originally proposed in [28] for Einstein gravity.

Gravitational and scalar self-interactions have fundamentally different behaviour. Let us first consider the familiar case of Einstein gravity. The corresponding Newtonian limit is known to work well for most systems of interest. Effects due to graviton self-interactions are suppressed because of the specific form of the self-coupling of the graviton. The three-graviton vertex, for instance, is of the form $\partial^2 h^3$ (where h denotes the graviton with generic polarisation indices and ∂ a generic spacetime derivative) and the corresponding correction to the Newtonian potential is proportional to $1/r^2$ [28]. The correction is thus suppressed with respect to the leading Newtonian contribution by a factor r_s/r (where r_s is the Schwarzschild radius of the massive object giving rise to the gravitational field). These corrections are indeed negligible for objects that are sufficiently distant or diffuse.

The scenario is different for the case of scalar self-interactions, as a scalar field can have a *non-derivative* ϕ^3 self-coupling. Two powers less of momentum result in two powers more of r in the self-interaction amplitude, or the effective potential, which in turn leads to a *logarithmic* correction to the “Newtonian” $1/r$ potential. This next-to-leading order correction overcomes the Newtonian potential at large enough distances. So ϕ^3 self-interactions may significantly change the long-range behaviour of the scalar field, in contrast to the gravitational case. This argument is explained quantitatively in Sect. 3.

Field theory techniques like Feynman diagrams were first applied to gravity and scalar tensor theories in [29], where *derivative* self-interactions of the type $\partial^2 \phi^n$ (for $n = 3, 4$) were considered. Derivative self interactions were also studied in other models equivalent to tensor scalar theories, such as the DGP and massive gravity models mentioned above, where the leading-order extra-scalar self-interactions are of the type $\partial^n \phi^3$, with $n = 4, 6$ respectively.

This article is organised as follows. In Sect. 2, we recall the basic ingredients and known result of chameleon models. In Sect. 3, we study the effect of self-interactions on the chameleon mediated potential for a generic choice of parameters. We show that the self-interactions may *grow* over astrophysical distances, even though for realistic values of the physical parameters, self-interactions do not lead to any dramatic phenomenological consequences. We finally conclude in Sect. 4.

2 Chameleon in brief

Consider the following action

$$\mathcal{S} = s \int d^4x \sqrt{-g} \left[\frac{\mathcal{R}}{16\pi G_N} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + \int d^4x \sqrt{-g} \mathcal{L}_{\text{matter}} \left(\psi^{(i)}, g_{\mu\nu}^{(i)} \right), \quad (1)$$

with signature $(-, +, +, +)$ for the metric $g_{\mu\nu}$, $g = \det g_{\mu\nu}$, \mathcal{R} the Ricci scalar, G_N the Newton constant, ϕ the chameleon field subject to the fundamental potential $V(\phi)$ and coupled to matter fields $\psi^{(i)}$ via the modified metric $g_{\mu\nu}^{(i)} \equiv e^{\beta_i \phi / M_{Pl}} g_{\mu\nu}$. We also define $M_{Pl} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV.

Let the fundamental potential be an inverse power-law of the form

$$V(\phi) = M^4 (M/\phi)^\alpha, \quad (2)$$

with α positive and M the fundamental mass scale of the problem (possibly much smaller than M_{Pl}). The specific form of the potential, however, is not crucial for the result we wish to discuss, but we nevertheless require it to be a decreasing function without a minimum. The chameleon couples to the trace of the energy momentum tensor of matter $T^{(i)} \equiv g^{\mu\nu} T_{\mu\nu}^{(i)}$. For non relativistic matter, we can safely replace $T^{(i)} \simeq -\rho$, where ρ is the energy density in the i -th particle species. We consider only one particle species and henceforth drop the index i . This coupling induces an effective potential

$$V_{\text{eff}}(\phi) = V(\phi) + \rho e^{\beta\phi/M_{Pl}} = M^4 \left(\frac{M}{\phi} \right)^\alpha + \rho e^{\beta\phi/M_{Pl}}. \quad (3)$$

Use of such an inverse power-law as the fundamental potential is an assumption, but qualitatively similar potentials appear in supergravity compactifications inspired by superstring constructions [30]. The minimum of the effective potential, defined by $V_{\text{eff},\phi}(\bar{\phi}) = 0$, is approximately given by

$$\bar{\phi} \simeq M \left(\frac{\alpha}{\beta} \frac{M_{Pl} M^3}{\rho} \right)^{1/(\alpha+1)}. \quad (4)$$

We consider a range of parameters for which this formula applies. In particular, for this formula to hold $\bar{\phi}$ must be sub-Planckian, which is the case if M is small enough and ρ is large enough, i.e.

$$M < \left(\frac{\beta^\alpha}{\alpha} M_{Pl}^\alpha \rho \right)^{\frac{1}{\alpha+4}}. \quad (5)$$

For instance, $\rho = 1 \text{ g/cm}^3 \simeq (5 \times 10^{-2} \text{ MeV})^4$, $\alpha=2$ relation (5) implies $M < 3 \text{ TeV}$.

Table 1 Value of the parameters for different densities ($\alpha = \beta = 1$, $M = 1$ eV)

	$\rho(\text{g/cm}^3)$	$m_\phi(\text{eV})$	λ_C (m)	$-g_3$	λ	$\bar{\phi}/M$
Earth	5.5	1.4×10^{-6}	0.14	6.4×10^{-16}	2.6×10^{-19}	9.8×10^3
Atmosphere	1.3×10^{-3}	2.8×10^{-9}	72	3.6×10^{-23}	2.2×10^{-28}	6.4×10^5
Interplanetary space	10^{-24}	4×10^{-25}	5×10^{17}	2×10^{-65}	4×10^{-81}	2×10^{16}

For comparison, $1\text{pc} \simeq 3 \times 10^{16}$ m

The effective potential, Taylor expanded around the minimum, is

$$V_{\text{eff}}(\phi) = \frac{m_\phi^2}{2} (\phi - \bar{\phi})^2 + \frac{g_3 M}{3!} (\phi - \bar{\phi})^3 + \frac{\lambda}{4!} (\phi - \bar{\phi})^4 + \sum_{n>4} \frac{g_n}{n! M^{n-4}} (\phi - \bar{\phi})^n, \quad (6)$$

where the following ρ -dependent parameters have been defined: the mass m_ϕ , the dimensionless trilinear coupling g_3 , the quartic coupling $g_4 \equiv \lambda$ and the generic dimensionless n -linear coupling g_n . Such parameters are related to derivatives of the effective potential evaluated at $\bar{\phi}$ and they are approximately given by

$$\begin{aligned} m_\phi^2 &\simeq (\alpha + 1) M^2 \left(\frac{\beta \rho}{M^3 M_{Pl}} \right)^{\frac{\alpha+2}{\alpha+1}} \exp(\beta \bar{\phi}/M_{Pl}), \\ g_3 &\simeq -(\alpha + 1)(\alpha + 2) \left(\frac{\beta \rho}{M^3 M_{Pl}} \right)^{\frac{\alpha+3}{\alpha+1}} \exp(\beta \bar{\phi}/M_{Pl}), \\ \lambda &\simeq (\alpha + 1)(\alpha + 2)(\alpha + 3) \left(\frac{\beta \rho}{M^3 M_{Pl}} \right)^{\frac{\alpha+4}{\alpha+1}} \exp(\beta \bar{\phi}/M_{Pl}). \end{aligned} \quad (7)$$

These parameters indeed depend on the local matter density. Setting $M = 1$ eV and $\alpha = \beta = 1$, typical values for the earth, the atmosphere and interplanetary space are summarised in Table 1.

For later reference g_n is given by

$$g_n = (-1)^n \frac{(\alpha + n - 1)!}{\alpha!} \left(\frac{M}{\bar{\phi}} \right)^{\alpha+n}, \quad (8)$$

where the factor involving the exponential of ϕ has been neglected. We will see in Sect. 3 that the interactions with $n \geq 5$ are of negligible phenomenological impact.

Following [9, 26], we determine the profile of the chameleon in the case of a spherically symmetric source of density ρ_c and radius R , surrounded by an environment of density ρ_∞ . As a first approximation of the long range behaviour of the chameleon in

the presence of massive objects, one can linearise the time-independent, spherically symmetric equation of motion by keeping only the quadratic term in the potential expansion (6), to obtain

$$\phi'' + \frac{2}{r}\phi' - m_c^2(\phi - \phi_c) = 0, \quad (9)$$

where m_c , ϕ_c respectively denote the value of m_ϕ , $\bar{\phi}$ inside the source and we have neglected the curvature of the space-time induced by the source. This approximation is usually referred to as the “thin shell” approximation in the literature. In the $m_c R \gg 1$ case, the “thin shell” approximation is appropriate, since ϕ is close to the minimum within most of the source and only becomes significant for a thin shell close to its edge, as discussed below. Following standard notation, we introduce ϕ_∞ , the field value at the minimum of the effective potential for $\rho = \rho_\infty$ (outside the source). In particular, we consider the case in which the environment, outside the source, is endowed with such a small density that the Compton wavelength of the corresponding chameleon is much greater than the length scales of interest ($m_\infty r \ll 1$).

The solution to the linearised equation (9) inside the source object, and the corresponding solution outside is

$$\begin{aligned} \phi(r < R) &= A \frac{\sinh(m_c r)}{r} + \phi_c, \\ \phi(r > R) &= B \frac{e^{-m_\infty(r-R)}}{r} + \phi_\infty, \end{aligned} \quad (10)$$

where the integration constants are fixed by requiring the solution and its derivative to continuously match at the boundary $r = R$, giving

$$\begin{aligned} A &= \left[\frac{1 + m_\infty R}{m_c R \coth(m_c R) + m_\infty R} \right] \frac{R(\phi_\infty - \phi_c)}{\sinh(m_c R)}, \\ B &= \left[\frac{m_c R \coth(m_c R) - 1}{m_c R \coth(m_c R) + m_\infty R} \right] R(\phi_c - \phi_\infty). \end{aligned} \quad (11)$$

Other solutions can be obtained as in [9, 26] using different approximations, but here Eq. (10) encompasses the relevant physics for macroscopic orbiting bodies, like planets and stars. For instance in the case $\phi(r) > \phi_c$ inside the source (i.e. $r < R$), the effective potential can be approximated by its increasing branch leading to the approximate equation of motion

$$\phi'' + \frac{2}{r}\phi' + \frac{\beta\rho}{M_{Pl}} = 0, \quad (12)$$

and retaining the approximate form (9) of the equation of motion *outside* the source of total mass M_c , one has the solution

$$\begin{aligned}\phi(r < R) &= -\frac{\beta\rho_c r^2}{6M_{Pl}} + \text{const}, \\ \phi(r > R) &= \frac{\beta}{4\pi M_{Pl}} \frac{M_c e^{-m_\infty(r-R)}}{r} + \phi_\infty.\end{aligned}\quad (13)$$

This profile is a self-consistent solution for $\phi(r < R) > \phi_c$ and has been named the *thick shell* solution, occurring for $m_c R < 1$.

For objects of astrophysical interest, like planets and stars, the size of the source of the chameleon field is larger than its corresponding Compton wavelength, as can be checked from Table 1. Consequently we focus on Eq. (10), in which case only a *thin shell* at the surface of the object contributes to the overall chameleon field. In this case the solution outside the source can be rewritten as

$$\phi(r > R) \simeq -\frac{\beta_{\text{eff}}}{4\pi M_{Pl}} \frac{M_c e^{-m_\infty(r-R)}}{r} + \phi_\infty \quad m_c R \gg 1, \quad (14)$$

where

$$\beta_{\text{eff}} = \frac{3\phi_\infty M_{Pl}}{\rho_c R^2}. \quad (15)$$

The standard Newtonian potential is analogous to Eq. (14), with β_{eff} replaced by unity, thus the suppression of the chameleon coupling is conveniently parameterized by β_{eff} . For instance, considering the earth as a sphere of radius 6.4×10^6 m with homogeneous density $\rho_\oplus \simeq 5.5 \text{ g/cm}^3$ immersed in the galactic medium made of dark matter and baryonic gas with density $\rho_G \simeq 10^{-24} \text{ g/cm}^3$, then $\beta_{\text{eff}} \simeq 5 \times 10^{-3}$ for $M = 1 \text{ eV}$, and $\alpha = \beta = 1$.

The thin-shell solution has the beneficial effect of suppressing the otherwise phenomenologically dangerous coupling to matter, but it does so in a non-universal manner. The chameleon couples to matter not only through the total mass of the object, but also through parameters such as the radius of the source. This peculiar coupling leads to violation of the equivalence principle, even in its weak form.

There is nevertheless one major caveat in the above analysis in obtaining the solutions, due to the approximations that have been made. In the thin-shell case, the effective potential is approximated by its quadratic expansion around the minimum $V_{\text{eff}}(\phi) \simeq m^2(\phi - \phi_c)^2/2$. The effect of higher order terms on the solution outside the source should be checked, however, since they may be important for the lowest order solution (10) at the boundary of the source. Naively substituting the solution (10) into the potential expansion (6), one realizes that the linear term in the equation of motion (the mass term) is dominated by the tri-linear and quartic interaction term for $r \lesssim \frac{1}{2}(\alpha + 2)R$ and $r \lesssim \sqrt{\frac{1}{3!}(\alpha + 2)(\alpha + 3)}R$, respectively, which are both clearly outside the source. In general, the self-interaction terms ϕ^n in the expansion (6) for $n \geq 2$ dominate at distances

$$r \lesssim \left(\frac{1}{(n-1)!} \frac{(\alpha + n - 1)!}{(\alpha + 1)!} \right)^{\frac{1}{n-2}} R. \quad (16)$$

The fact that all of the higher order terms overcome the linear term indicates the need for a more thorough analysis.

In [23], the effect of a $\lambda\phi^4$ interaction is studied using approximate analytic methods. The resulting profile for the chameleon in the thin shell case is still $1/r$ outside the source $r > R$ (and within the Compton wavelength of the chameleon $m_\infty r \ll 1$), but the effective coupling reduces to

$$\beta_{\text{eff}} \simeq \frac{M_{Pl}}{M} \lambda^{-1/2} \quad \text{if } \lambda > \left(\frac{M_{Pl}}{\beta M} \right)^2, \quad (17)$$

which occurs for large enough λ . This is a non-perturbative effect which cannot be reproduced in a perturbative analysis. In [26] an approximate analytic analysis of the non-linear regime is performed, and a result equivalent to (14) is obtained (see next to last eq. in sec. IIID of [26]). They show that the full form of the potential does not substantially alter the solution close to the surface of the source, where the matching between the inner and the outer solutions in (10) is made. The argument of [26] is roughly as follows. Since, at large distances, the solution can only be of the type of the second equation in (10) and since $\phi(r) > 0$, we must have $-\phi_\infty R < B < 0$ so that $\phi(r) > \phi_{\text{critical}}(r) \equiv \phi_\infty (1 - R e^{-m_\infty(r-R)}/r)$. The limiting value $\phi_{\text{critical}}(r)$ coincides with the approximate solution (14). If, on the other hand, $\phi(r) \gg \phi_{\text{critical}}(r)$ outside of the body, then one would simply be in the case leading to solution (13).

In Sect. 3 we use a perturbative effective field theory method borrowed from [28] which gives systematic estimates of scalar field self-interactions, particularly relevant to determine the corrections to the chameleon profile outside a source where the potential is very shallow. We focus on the *thin shell* solutions as it is the astrophysically relevant solution.

3 Corrections to the potential from chameleon self-interactions

We have so far recalled how the *thin-shell* solution is obtained and mentioned its corrections treated in the literature. We present here another tool to compute the chameleon profile beyond the linearised approximation. We adopt an effective approach in which sources are considered to be point-like and coupled to a massive chameleon with strength β_{eff} , giving the original chameleon-mediated potential (14). In addition to the mass term, the chameleon field is subject to the effective potential V_{eff} of Eq. (6). To simplify the problem we initially truncate the effective potential V_{eff} to cubic order. The effect of higher order interactions in V_{eff} are considered later in this section.

We study the following Lagrangian

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \left(\partial_\mu \phi \partial^\mu \phi + m_\phi^2 \phi^2 + \frac{g_3 M}{3} \phi^3 \right) + \beta_{\text{eff}} \frac{M_c}{M_{Pl}} \phi \int dt. \quad (18)$$

The ϕ field is redefined so that its minimum is at $\phi = 0$ and a constant term in the Lagrangian is neglected. With $g_3 = 0$, this Lagrangian reproduces precisely the potential of the thin shell solution (14) outside the source. Following [28], where

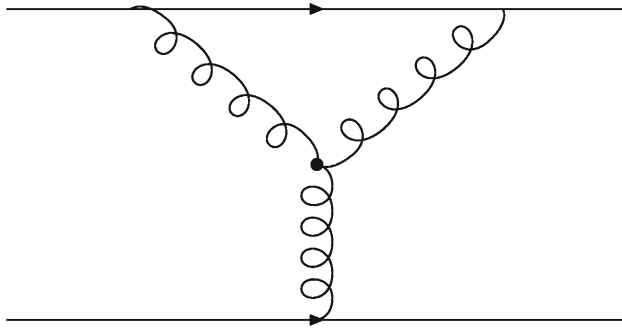


Fig. 1 Feynman diagram displaying the contribution to the effective potential proportional to the tri-linear interaction $g_3 M \phi^3$

effective field theory methods for gravity are discussed, the problem is treated non-relativistically, thus splitting the kinetic term of (18) into

$$\partial_\mu \phi \partial^\mu \phi = \delta_{ij} \partial_i \phi \partial_j \phi - \dot{\phi}^2, \quad (19)$$

and treating the time derivative as an interaction term. In terms of Fourier transformed functions

$$\phi_{\mathbf{k}}(t) \equiv \int \frac{d^3 x}{(2\pi)^3} \phi(t, \mathbf{x}) e^{i\mathbf{k} \cdot \mathbf{x}}, \quad (20)$$

for the ϕ -propagator we have

$$\langle \phi_{\mathbf{q}}(t) \phi_{\mathbf{k}}(0) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{q} + \mathbf{k}) \delta(t) \frac{1}{k^2 + m_\phi^2}, \quad (21)$$

where the four-momentum $k^\mu = (k^0, \mathbf{k})$, with $k \equiv \sqrt{\mathbf{k} \cdot \mathbf{k}}$.

To account for the effect of the tri-linear interaction on the chameleon potential, the amplitude represented by the Feynman diagram in Fig. 1 has to be computed.

The source masses are M_{c1} , M_{c2} with trajectories $\mathbf{x}_1(t)$, $\mathbf{x}_2(t)$ respectively, see [27]. The 3-point function of the ϕ -field is

$$\begin{aligned} \langle T(\phi(x_1)\phi(x_2)\phi(x_3)) \rangle &= 3!(-ig_3 M)\delta(t_1 - t_2)\delta(t_2 - t_3) \\ &\times \int \prod_{r=1}^3 \frac{d^3 k_r}{(2\pi)^3} e^{i\mathbf{k}_r \cdot \mathbf{x}_r} (2\pi)^3 \delta^{(3)}\left(\sum_{r=1}^3 \mathbf{k}_r\right) \prod_{r=1}^3 \frac{-i}{k_r^2 + m_\phi^2}. \end{aligned} \quad (22)$$

Then the diagram in Fig. 1 can then be computed to give a contribution to the effective action [27]

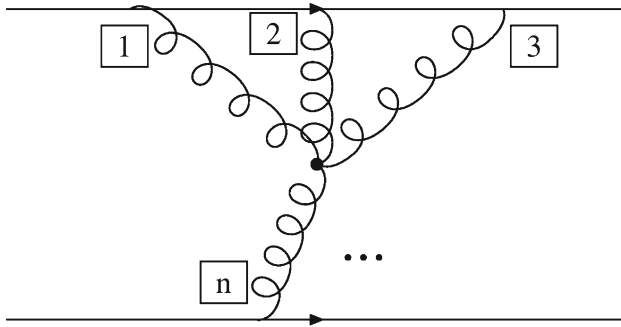


Fig. 2 Feynman diagram displaying the generic contribution to the effective potential proportional to the n -point interaction $g_n M^{4-n} \phi^n$

$$\begin{aligned} \text{Fig. 1} &= \frac{1}{12} \left(\frac{-i M_{c1}}{M_{Pl}} \right)^2 \left(\frac{-i M_{c2}}{M_{Pl}} \right) \int dt_1 dt'_1 dt_2 \langle T (\phi(x_1) \phi(x'_1) \phi(x_2)) \rangle \\ &= -i g_3 \frac{\beta_{\text{eff}}^3 M M_{c1}^2 M_{c2}}{M_{Pl}^3} \int dt \log(m_\phi |\mathbf{x}_1 - \mathbf{x}_2|). \end{aligned} \quad (23)$$

In the computation, the mass term in the propagator is neglected, as we consider distances (here and in the rest of this section) $r \ll 1/m_\phi$, but it is reinserted at the end of (23) as an infrared regulator. One immediately notices that this logarithmic contribution to the effective potential *grows* with distance, which eventually overcomes the lowest order solution (10) is a distance at r_* .

When the body is taken to be the earth, this distance corresponds to

$$r_* \simeq \frac{1}{g_3} \frac{M_{Pl}}{\beta_{\text{eff}} M_c M}, \quad (24)$$

i.e. $r_* \simeq 10^6$ Mpc (larger than the Hubble radius) for $M = 1$ eV and $R = R_\oplus$, $\rho = \rho_\oplus$, so it is negligible for reasonable distances.

For higher order interactions, it is not necessary to perform the actual computation of the relevant Feynman diagram Fig. 2. Using effective field theory methods, we can indeed estimate the scaling of the relative amplitude for the contribution to the effective potential mediated by all the other $g_n M^4 (\phi/M)^n$ interactions.

In effective field theory, one usually has an expansion in a perturbative parameter. For instance, in general relativity, post-Newtonian computations for two bodies gravitating around each other have the typical velocity v of the system as the expansion parameter (n -th PN computation means v^{2n} correction to the Newtonian result). In our case, as we will see, we have a proliferation of dimensionless scales, so it is not possible to identify a single expansion parameter, however the method of [28] can still be applied to assess the strength of different contributions to the effective potential.

The effective action is computed perturbatively by applying systematic power counting rules, relying on the basic assumption that Newtonian gravity is still responsible for the leading interaction, so that the virial relation $v^2 \sim G_N M_c / r$ holds.

Each insertion on the world-line of a source, propagator, ϕ vertex, requires one of the following factors

- $\beta_{\text{eff}} dt d^3k M_c/M_{Pl}$ for a particle–chameleon vertex
- $g_n M^{4-n} dt (d^3k)^n \delta^{(3)}(k)$ for each n - ϕ vertex
- $\delta(t) \delta^{(3)}(k) \times 1/k^2$ for each propagator.

In standard Einstein gravity, two gravitating bodies exchange gravitons mediating the gravitational potential with momenta $k^\mu = (k_0, \mathbf{k})$, where $k^0 \sim v/r$ and $k \sim 1/r$ [28]. One can then assign the scaling $x^0 \sim r/v$, $x \equiv |\mathbf{x}| \sim r$, $k \sim 1/r$ and consequently $\delta^{(3)}(k) \sim r^3$, $d^3k \sim 1/r^3$, $\delta(t) \sim v/r$, $dt \sim r/v$.

These rules can be applied to a diagram contributing to the effective potential between two massive objects due to the exchange of ϕ -fields and involving an n - ϕ vertex ($n > 2$), so the scaling becomes

$$\text{Fig. 2} \sim g_n \beta_{\text{eff}}^n M^{4-n} \left(\frac{M_c}{M_{Pl}} \right)^n \frac{r^{4-n}}{v}. \quad (25)$$

We recall that the simple one graviton exchange scales as [28]

$$\left(\frac{M_c}{M_{Pl}} \right)^2 \frac{t}{r} \simeq M_c v r \equiv L, \quad (26)$$

where the virial relation $M_c/(M_{Pl}^2 r) \sim v^2$ has been used. Using both (26) and the same virial relation in the chameleon amplitude scaling (25) one obtains

$$\text{Fig. 2} \sim g_n \beta_{\text{eff}}^n L \left(\frac{r_s}{l_{Pl}} \right)^2 \left(\frac{r_s}{r} \right)^{n-4} \left(\frac{M}{M_{Pl}} \right)^{4-n}, \quad (27)$$

where $r_s \equiv 2G_N M_c$ is the Schwarzschild radius of the source and the Planck length $l_{Pl} \equiv M_{Pl}^{-1}$ has been introduced. The proliferation of dimensionless ratios (r_s/r , M/M_{Pl} and M_c/M_{Pl}) renders this result less immediate to interpret, but the substitution of actual values for physical parameters provides insight as to what the scaling (27) means. Let us observe that L is the scaling of the action involving the Newtonian potential ($\sim dt G_N M_c^2/r$), and $\beta_{\text{eff}}^2 L$ is the scaling of the contribution to the effective action obtained by a diagram analogous to Fig. 2 but with just one ϕ -propagator. Apart from $\beta_{\text{eff}}^2 L$, there are in Eq. (27) extra terms involving the ratios $r_s/r < 1$, $M/M_{Pl} < 1$ and $r_s/l_{Pl} > 1$. To evaluate the importance of the diagram involving the n - ϕ vertex, like in Fig. 2 for the case of two orbiting bodies, an estimation of the parameters is necessary. Numerical values are summarised by

$$\begin{aligned} \left(\frac{r_s}{l_{Pl}} \right)^2 &\simeq \left(4 \cdot 10^{37} \times \frac{M_c}{M_\odot} \right)^2, \\ \frac{r_s}{r} &\simeq 2 \cdot 10^{-8} \times \frac{M_c}{M_\odot} \left(\frac{r}{1\text{AU}} \right)^{-1}, \end{aligned}$$

$$\frac{M}{M_{Pl}} \simeq 4 \cdot 10^{-28} \times \frac{M}{1eV},$$

$$\beta_{\text{eff}} \simeq 2.2 \cdot 10^{-6} \times \left(\frac{\phi_{\infty}}{10^{16} \text{eV}} \right) \left(\frac{R}{R_{\odot}} \right) \left(\frac{M_c}{M_{\odot}} \right)^{-1}. \quad (28)$$

For reasonable values of the parameters, such corrections are less and less relevant as n increases, and the resulting contribution to the two-body potential goes with distance as $1/r^{n-3}$. The case of the amplitude in Eq. (23) corresponds to $n = 3$, i.e. a logarithmic potential, or as Eq. (27) shows, a potential whose r dependence displays one power more than the $1/r$, Newtonian, usual behaviour of single particle exchange. For $n = 4$ the amplitude is a times the contribution from the diagram with a single chameleon exchange, where a is given by

$$a \equiv \beta_{\text{eff}}^2 \lambda (r_s / l_{Pl})^2. \quad (29)$$

Taking the value for λ in interplanetary medium from Table 1, we see that a becomes a strong suppression factor. To answer the issue mentioned above Eq. (16), it is of little importance that the higher order terms in the expansion are as important as the mass term near the surface of the body: as we are considering here the case $r \ll 1/m_{\infty}$ the potential is simply negligible for the determination of the chameleon profile in the region well within the Compton wavelength, where the Yukawa suppression has not yet taken place. Going from a diagram with a n -point interaction vertex to one with a $n + 1$ boils down to multiplying the amplitude by a factor given by

$$\frac{g_{n+1}}{g_n} \beta_{\text{eff}} \left(\frac{r_s}{r} \right) \left(\frac{M_{Pl}}{M} \right) \simeq \beta_{\text{eff}} \left(\frac{r_s}{r} \right) \left(\frac{M_{Pl}}{\phi_{\infty}} \right), \quad (30)$$

which again is smaller than unity for reasonable parameters, as can be estimated from Table 1 and (28). The perturbative series is then under full control.

The kinetic term and the potential of the chameleon field, including all non-linearities, contribute to the energy-momentum tensor which, in turn, affect the background gravitational field via the Einstein equations. It is therefore necessary to verify that this chameleon induced backreaction on the gravitational field is negligible compared to the background gravitational field. This verification is done by comparing the respective chameleon and gravitational potentials as follows. Let T be the generic entry of the energy-momentum tensor of the chameleon field and h the metric change due to its effect. We then obtain from Einstein equations:

$$h'' \sim 8\pi G_N T \sim \frac{\phi'^2 + V(\phi)}{M_{Pl}^2} \sim \frac{\phi'^2 + m_{\infty}^2 (\phi - \phi_{\infty})^2}{M_{Pl}^2}$$

$$\sim \beta_{\text{eff}}^2 \frac{r_s^2}{r^4} + \beta_{\text{eff}}^2 m_{\infty}^2 \left(\frac{r_s}{r} \right)^2, \quad (31)$$

where only the contribution of the quadratic part of the chameleon potential has been considered. Analogous reasoning behind Eq. (30) leads to the conclusion that the

contribution from higher order chameleon self-interactions are sub-dominant with respect to the $m_\infty^2 (\phi - \phi_\infty)^2$ term. For $r < m_\infty^{-1}$, the term proportional to $1/r^2$ is sub-dominant with respect to the one proportional to $1/r^4$, thus leading to the estimate $h \sim \beta_{\text{eff}}^2 r_s^2 / r^2$, whose effect is suppressed by a small factor β_{eff}^2 compared to the first order Post-Newtonian correction to the gravitational potential and a factor $\beta_{\text{eff}} r_s / r$ compared to the leading chameleon solution. In the opposite regime $r > m_\infty^{-1}$, the Yukawa suppression takes place. We have thus verified that the metric backreaction due to the chameleon is indeed negligible compared to the background solution.

Extra long-range forces have to fulfill experimental constraints, as Einstein gravity is known to work perfectly well. For the chameleon, such constraints have already been taken into consideration in several articles, see e.g. [26, 31], in the following we quote the ones relevant for orbiting bodies in the effective theory defined by Eq. (18). A different acceleration of the moon and the earth towards the sun $\eta_{\oplus-m}$ would have been detected, had it exceeded the fractional value of $\eta_{\oplus-m}^{(exp)} \sim 10^{-13}$ [3]. Violating the WEP, the chameleon mediated force induces a differential acceleration $\eta_{\oplus-m}^{(cham)}$ for the earth and the moon toward the sun (normalised to the ordinary acceleration towards the sun at 1 AU), of strength roughly given by

$$\begin{aligned} \eta_{\oplus-m}^{(cham)} &\simeq \beta_{\text{eff}\odot} |\beta_{\text{eff}\oplus} - \beta_{\text{eff}m}| \\ &\simeq 16\pi^2 \frac{\phi_\infty^2 R_\odot R_m}{M_\odot M_m / M_{Pl}^2} \simeq 3.6 \cdot 10^{-7} \times \left(\frac{M}{\text{eV}} \right)^{\frac{2\alpha+4}{\alpha+1}}, \end{aligned} \quad (32)$$

which evades the bound on $\eta_{\oplus-m}$ for $M \lesssim 3 \times 10^{-7 \frac{\alpha+1}{2(\alpha+4)}}$ eV (where α and β have been set to 1 elsewhere than in the exponent).

Considering other astrophysical constraints from orbiting bodies, usually such bounds are expressed in terms of the Brans–Dicke parameter ω_{BD} , defined by the action S_{BD} ruling the dynamics of the Brans–Dicke scalar field Φ ,

$$\begin{aligned} S_{BD} = S_\Phi + S_m &= \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g} \left[e^\Phi \left(R - \omega_{BD} (\partial\Phi)^2 \right) \right] \\ &+ \int d^4x \sqrt{-g} \mathcal{L}_m(\psi^i, g_{\mu\nu}). \end{aligned} \quad (33)$$

Once Φ is canonically normalised, i.e. $\Phi^c \equiv \Phi / (M_{Pl} \omega_{BD}^{1/2})$, and the metric rescaled by a factor $e^{-\Phi}$, Eq. (33) gives a coupling to matter

$$S_m = \frac{M_c}{2M_{Pl} \omega_{BD}^{1/2}} \Phi^c \int dt. \quad (34)$$

Present bounds from Cassini spacecraft [12], for instance, give $\omega_{BD} > 4 \times 10^4$ [3], translating into $\beta_{\text{eff}} < 3 \times 10^{-3}$ for the chameleon, which is easily evaded by allowing a small enough M (e.g. $M < \text{few eV}$ in the case of the earth).

4 Conclusions

We have given a systematic analysis of the corrections to the chameleon mediated potential due to non-derivative n -th order self-interactions, performed through the implementation of the effective field theory method originally proposed in [28] for General Relativity, in the case of weakly self interacting bodies (which include stars and planets but not black holes). The analysis of the effects of self interaction is crucial for understanding if the result for the free field can be extended to the fully interacting case. Trilinear interactions have been shown to be potentially dangerous, as their contribution to the potential grows with distance with respect to the lowest order effect, but for ordinary values of the parameters at play, such contributions are actually harmless. Contributions from higher order interactions decrease faster with distance and they are also negligible. We have presented a thorough perturbative analysis of such effects with the help of effective field theory methods and the powerful tool of Feynman diagrams. Once the relevant scales in the problem are identified, even if they are multiple, as in this case, it is possible to set a perturbative expansion which allows for the assessment of different diagrams.

Acknowledgments This work is supported by the Fonds National Suisse. The work of R.S. is supported by the Boninchi foundation. The authors wish to thank M. Maggiore for discussions and support.

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